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VECTOR AUTOREGRESSIVE MODELS USING “R”

Keywords

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Abstract

Multivariate data analysis in the context of autoregressive models has evolved as a standard instrument in econometrics. In present, there are developed packages available in R for estimating time series models; one of the most useful package is vars (Pfaff, 2008) containing functions for diagnostic testing, estimation of a restricted models, prediction, causality analysis, impulse response analysis and forecast error variance decomposition. Using the examples provided in the vars vignette, the authors tried to obtain results for the different methods and functions on the base of macroeconomic data set for Romania.

1. INTRODUCTION

In 1980, Sims criticized for the first time the simultaneous equation literature for the ad hoc restrictions needed for identification and for the classification of exogenous and endogenous variables in the system (Baltagi, 2011). In the next period, the multivariate data analysis in the context of vector autoregressive models became a very important tool in applied statistics and econometrics. An argument is the reason that the statistical tests are frequently used in determining inter-dependencies and dynamic relationships between variables, conducting to a fast developing of methodology. Vector Autoregressive models (noted VAR) explain the endogenous variables on the base of their own history, apart from deterministic regressors, as an autoregressive function of their lagged values. Also, the VAR models can include some exogenous variables as like trends and seasonal dummies, but it does not have to classify variables as endogenous or exogenous. The Vector Autoregressive models explain the endogenous variables solely by their own history, apart from deterministic regressors, as

an autoregressive function of their lagged values. Also, the VAR models could include certain exogenous variables like trends and seasonal dummies, but it does not have to classify variables as endogenous or exogenous (Pfaff, 2008).

Thwartly, the structural vector autoregressive models, SVAR, allow the explicit modelling of contemporaneous interdependence between the left-hand side variables. Hence, these types of models try to bypass the shortcomings of VAR models. At the same time as Sims jeopardized the paradigm of multiple structural equation models laid out by the Cowles Foundation in the 1940s and 1950s, Granger (1981) and Engle and Granger (1987) endowed

With A_i are $(T \times T)$ coefficient matrices for $i = 1, \dots, p$ and u_t is a T -dimensional process with $E(u_t) = 0$ and time invariant positive definite covariance matrix $E(u_t u_t^K) = \Sigma_u$ (white noise).

One important characteristic of a VAR(p)-process is its stability. This explains

If the solution of the above equation has a root for $n = 1$, then either some or all variables in the VAR (p) - process are integrated of order one, i.e., $I(1)$. It might be the case of a cointegration between the variables. This instance can then be better analyzed in the context of a vector error correction model (VECM).

With:

$$\begin{matrix}
 \mathbf{y}_k \\
 \vdots \\
 \mathbf{y}_{k > p-1}
 \end{matrix}
 \begin{matrix}
 \mathbf{y}_k \\
 \vdots \\
 \mathbf{y}_{k > p-1}
 \end{matrix}$$

$$A = \begin{bmatrix}
 A_1 & A_2 & \dots & A_{p-1} & A_p \\
 I & 0 & \dots & 0 & 0 \\
 0 & I & \dots & 0 & 0 \\
 \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & \dots & I & 0
 \end{bmatrix}$$

Whereby the dimensions of the stacked vectors ζ_t and v_t is $(Tp \times 1)$ and the dimension of the matrix A is $(Tp \times Tp)$. If the module of the Eigen values of A are less than one, then the VAR(p) – this means the process is stable.

econometricians with a powerful tool for modelling and testing economic relationships, namely, the concept of co-integration.

VAR models have also been used to test the hypothesis that some variables do not have Granger cause to some other variables.

2. VECTOR AUTOREGRESSIVE MODELS

A basic VAR model consists of a set of T endogenous variables $y_k = (y_{k1}, \dots, y_{k2}, \dots, y_{kT})$ for $t = 1, \dots, T$. The VAR(p)-process is then defined as:

$$y_k = A_1 y_{k-1} + \dots + A_p y_{k-p} + u_t \quad (1)$$

that it generates stationary time series with time invariant means, variances and covariance structure, given sufficient starting values. This can be checked by evaluating the characteristic polynomial:

$$\det(I_T - A_1 z - \dots - A_p z^p) \neq 0 \text{ for } |z| \leq 1 \quad (2)$$

In practice, the stability of an empirical VAR (p) - process can be analyzed by considering the companion form and calculating the Eigen values of the coefficient matrix. A VAR(p) - process can be written as a VAR(1) - process:

$$\zeta_k = A \zeta_{k-1} + v_t \quad (3)$$

$$\begin{matrix}
 \mathbf{u}_k \\
 \mathbf{0} \\
 \mathbf{v}_k \\
 \vdots \\
 \mathbf{0}
 \end{matrix}
 \begin{matrix}
 \mathbf{u}_k \\
 \mathbf{0} \\
 \mathbf{v}_k \\
 \vdots \\
 \mathbf{0}
 \end{matrix}
 \quad (4)$$

For a given example of the endogenous variables y_1, \dots, y_K and sufficient presample values y_{-p+1}, \dots, y_0 , the coefficients of a VAR(p) - process can be estimated efficiently by least-squares applied separately to each of the equations.

Once a VAR(p) model has been estimated, the way is open for further analysis

in near future. A researcher might be interested in diagnostic tests, such as testing for the absence of autocorrelation, heteroscedasticity or non-normality in the error process. Also he might be interested further in causal inference, forecasting and/or diagnosing the empirical $y_t = \Phi_0 u_t + \Phi_1 u_{t-1} + \Phi_2 u_{t-2} + \dots$

With $\Phi_0 = I_K$ and Φ_s can be computed recursively according to:

For $s = 1, 2, \dots$,
 Whereby $A_j = 0$ for $j > p$.

Finally, the forecasts for horizons $h \geq 1$ of an empirical VAR(p)-process can be generated recursively according to:

Where $y_{K+j|K} = y_{K+j}$ for $j \leq 0$. The forecast error covariance matrix is given as:

$$\text{Cov} \begin{matrix} y_{K-1} > y_{K-1|K} \\ \vdots \\ y_{K-h} > y_{K-h|K} \end{matrix} \begin{matrix} \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ N & W_1 & \mathbf{I} & \mathbf{0} \\ & \vdots & & \ddots \\ W_{h>1} & W_{h>2} & \dots & \mathbf{I} \end{matrix} \otimes \mathbf{I}_h = \begin{matrix} \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ W_1 & \mathbf{I} & & \mathbf{0} \\ \vdots & & \ddots & \mathbf{0} \\ W_{h>1} & W_{h>2} & \dots & \mathbf{I} \end{matrix} \quad (7)$$

and the matrices Φ_i are the empirical coefficient matrices of the Wold moving average representation of a stable VAR(p)-process as shown above. The operator \otimes is the Kronecker product.

VAR models are used for multivariate time series. The structure is that each variable is a linear function of past lags of itself and past lags of the other variables. Each variable is a linear function of the lag 1 values for all variables in the set.

In a VAR(2) model, the lag 2 values for all variables are added to the right side of the equations. In the case of three s -variables there would be six variables on the right side of each equation, three lag 1 variables and three lag 2 variables.

In general, for a VAR (p) model, the first p lags of each variable in the system would be used as regression predictors for each variable.

VAR models are a specific case of more general vector autoregressive moving-average models (VARMA). A VARMA model for multivariate time series includes the VAR structure above along with moving average terms for each variable. These are special cases

model's dynamic behavior, i.e., impulse response functions and forecast error variance decomposition. The latter two are based upon the Wold moving average decomposition for stable VAR(p)-processes, which is defined as:

$$\Phi_s = \sum_{j=1}^s \Phi_{s-j} A_j \quad (5)$$

$$y_{K+h|K} = A_1 y_{K+h-1|K} + \dots + A_p y_{K+h-p|K} \quad (6)$$

As an example, suppose that we measure three different time series variables, denoted by $s_{t,1}$, $s_{t,2}$, and $s_{t,3}$.

The vector autoregressive model of order 1, denoted as VAR(1), is as follows:

$$s_{t,1} = \alpha_1 + \beta_{11} s_{t-1,1} + \beta_{12} s_{t-1,2} + \beta_{13} s_{t-1,3} + w_{t,1} \quad (8)$$

$$s_{t,2} = \alpha_2 + \beta_{21} s_{t-1,1} + \beta_{22} s_{t-1,2} + \beta_{23} s_{t-1,3} + w_{t,2} \quad (9)$$

$$s_{t,3} = \alpha_3 + \beta_{31} s_{t-1,1} + \beta_{32} s_{t-1,2} + \beta_{33} s_{t-1,3} + w_{t,3} \quad (10)$$

of ARMAX models allowing for the addition of other explicative variables that are outside the multivariate set of principal interest variable.

3. DATA ANALYSIS USING R

A preliminary data analysis is conducted by displaying the summary statistics of the series involved. After that, unit root tests were conducted by applying the Augmented Dickey-Fuller test regressions to the series (ADF test).

We denote the following:

prod = log (GDP/employment);

e = log (employment);

u = unemployment rate;

rw = real wages (log of the real wage index).

The data sets used are from the period 2005-2012, quarterly, for GDP, employment and unemployment rates and real wages (see the *Annex*). The result of the ADF test is presented below, first with trend, then with drift:

```
> adf1 <- summary(ur.df(dataset[,  
"prod"], type = "trend", lags = 2))  
> adf1
```

Augmented Dickey-Fuller Test Unit Root Test
Test regression trend

Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt +
z.diff.lag)

Residuals:
Min 1Q Median 3Q Max
-0.069868 -0.019356 -0.004538 0.014875
0.071735

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.298387 0.929411 1.397
0.176
z.lag.1 -0.133653 0.098952 -1.351 0.190
tt 0.002229 0.002675 0.833 0.413
z.diff.lag1 0.054420 0.203514 0.267
0.792
z.diff.lag2 0.089021 0.201160 0.443
0.662

Residual standard error: 0.03228 on 23 degrees
of freedom
Multiple R-squared: 0.1605, Adjusted R-
squared: 0.01456
F-statistic: 1.1 on 4 and 23 DF, p-value:
0.3804

Value of test-statistic is: -1.3507 2.7364
1.8605

Critical values for test statistics:
1pct 5pct 10pct
tau3 -4.15 -3.50 -3.18
phi2 7.02 5.13 4.31
phi3 9.31 6.73 5.61

```
> adf2 <- summary(ur.df(diff(dataset[,  
"prod"]), type = "drift", lags = 1))  
> adf2
```

Augmented Dickey-Fuller Test Unit Root Test
Test regression drift

Call:
lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)

Residuals:
Min 1Q Median 3Q Max
-0.083811 -0.017178 -0.000067 0.009892
0.092719

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.018768 0.009159 2.049
0.05108 .
z.lag.1 -0.795067 0.257340 -3.090
0.00486 **
z.diff.lag -0.103986 0.190072 -0.547
0.58917

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05
'.' 0.1 ' ' 1

Residual standard error: 0.03337 on 25 degrees
of freedom
Multiple R-squared: 0.4723, Adjusted R-
squared: 0.4301
F-statistic: 11.19 on 2 and 25 DF, p-value:
0.0003389

Value of test-statistic is: -3.0896 4.8112

Critical values for test statistics:
1pct 5pct 10pct
tau2 -3.58 -2.93 -2.60
phi1 7.06 4.86 3.94

In the next step, an optimal lag length was
determined for an unrestricted VAR for a
maximal lag length of four.
First, the variables have to be reordered:

```
> Romania <- dataset[, c("prod", "e",  
"u", "rw")]  
> VARselect(Romania, lag.max = 4, type  
= "both")
```

\$selection
AIC(n) HQ(n) SC(n) FPE(n)
4 4 1 3

\$criteria
1 2 3 4
AIC(n) -2.439873e+01 -2.498365e+01 -
2.576438e+01 -2.637020e+01
HQ(n) -2.405623e+01 -2.441280e+01 -
2.496519e+01 -2.534268e+01

```

SC(n) -2.324688e+01 -2.306389e+01 -
2.307672e+01 -2.291464e+01
FPE(n) 2.611347e-11 1.636639e-11
1.010456e-11 1.064656e-11

```

According to the AIC and HQ, the lowest value (Jula, 2010), the optimal lag number is $p = 4$, while SC indicates $p = 1$ and FPE criterion indicates an optimal lag length of $p = 3$.

In the R code lines below is shown the estimation of the VAR(1), as well as the summary output and the diagram of fit for equation "e".

```

p1ct <- VAR(Romania, p = 1, type =
"both")
p1ct

```

VAR Estimation Results:

Estimated coefficients for equation prod:

```

Call:
prod = prod.l1 + e.l1 + u.l1 + rw.l1 + const +
trend

```

```

prod.l1  e.l1  u.l1  rw.l1  const
trend
0.747003575  0.494499694 -0.011836429 -
0.0087230 -5.1660943 0.0053210

```

Estimated coefficients for equation e:

```

Call:
e = prod.l1 + e.l1 + u.l1 + rw.l1 + const + trend

```

```

prod.l1  e.l1  u.l1  rw.l1  const
trend
0.263865079  0.587760510  0.009050756 -
0.1116373  4.674393 -0.00342123

```

Estimated coefficients for equation u:

```

Call:
u = prod.l1 + e.l1 + u.l1 + rw.l1 + const +
trend

```

```

prod.l1  e.l1  u.l1  rw.l1  const
trend
-4.56119165  -8.39440840  0.17017893
1.61923859 168.12486 0.09562047

```

Estimated coefficients for equation rw:

```

Call:
rw = prod.l1 + e.l1 + u.l1 + rw.l1 + const +
trend

```

```

prod.l1  e.l1  u.l1  rw.l1
const trend
0.56619561  0.7138821  0.0049096
0.4906261 -12.9409572 0.0017200

```

```
> summary(p1ct, equation = "e")
```

VAR Estimation Results:

Endogenous variables: prod, e, u, rw

Deterministic variables: both

Sample size: 30

Log Likelihood: 207.676

Roots of the characteristic polynomial:

```
0.8216 0.7226 0.2865 0.2865
```

Call:

```
VAR(y = Romania, p = 1, type = "both")
```

Estimation results for equation e:

```
e = prod.l1 + e.l1 + u.l1 + rw.l1 + const + trend
```

Estimate	Std. Error	t value	Pr(> t)
prod.l1	0.263865	0.105717	2.496
0.0198 *			
e.l1	0.587761	0.174061	3.377
0.0025 **			
u.l1	0.009051	0.006128	1.477
0.1527			
rw.l1	-0.111637	0.071780	-1.555
0.1330			
const	4.674393	2.824532	1.655
0.1110			
trend	-0.003421	0.001554	-2.202
0.0375 *			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01534 on 24 degrees of freedom

Multiple R-Squared: 0.5536, Adjusted R-Squared: 0.4607

F-statistic: 5.954 on 5 and 24 DF, p-value: 0.001024

Covariance matrix of residuals:

```

          prod  e  u  rw
prod  0.0009109 -0.0001877 -0.003576
0.0004013
e    -0.0001877  0.0002354 -0.001708 -
0.0001957
u    -0.0035759 -0.0017082  0.277226
0.0036219
rw   0.0004013 -0.0001957  0.003622
0.0009713

```

Correlation matrix of residuals:
 prod e u rw
 prod 1.0000 -0.4053 -0.2250 0.4267
 e -0.4053 1.0000 -0.2115 -0.4092

```
u -0.2250 -0.2115 1.0000 0.2207
rw 0.4267 -0.4092 0.2207 1.0000
```

```
> plot(p1ct, names = "e")
```

Diagram of fit and residuals for e

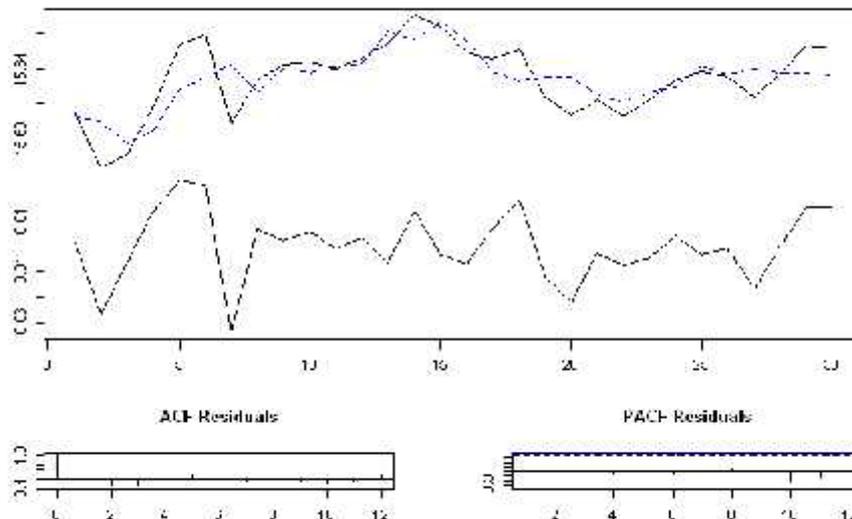


Figure 1. Diagram of fit and residuals for employment

Next, is shown how the diagnostic tests are conducted for the VAR(1) model.

```
> ser11 <- serial.test(p1ct, lags.pt = 16,
type = "PT.asymptotic")
> ser11$serial
```

Chi-squared = 3.1601, df = 4, p-value = 0.5314

\$Kurtosis

Kurtosis only (multivariate)

Portmanteau Test (asymptotic)

data: Residuals of VAR object p1ct
 Chi-squared = 3.0525, df = 4, p-value = 0.5491

data: Residuals of VAR object p1ct
 Chi-squared = 261.2736, df = 240, p-value = 0.165

```
> summary(ca.jo(Romania, type =
"trace", ecdet = "trend", K = 3, spec =
"transitory"))
```

```
> norm1 <- normality.test(p1ct)
> norm1$jb.mul
$JB
```

Johansen-Procedure
 Test type: trace statistic, with linear trend in cointegration

JB-Test (multivariate)

Eigenvalues (lambda):
 [1] 8.813106e-01 5.803882e-01
 4.306587e-01 2.006071e-01 -2.656685e-16

data: Residuals of VAR object p1ct
 Chi-squared = 6.2126, df = 8, p-value = 0.6234

Values of test statistic and critical values of test:

\$Skewness

	test	10pct	5pct	1pct
r <= 3		6.27	10.49	12.25 16.26
r <= 2		22.04	22.76	25.32 30.45
r <= 1		46.36	39.06	42.44 48.45

Skewness only (multivariate)

data: Residuals of VAR object p1ct

r = 0 | 106.03 59.14 62.99 70.05

Eigenvectors, normalised to first column:
(These are the cointegration relations)

	prod.l1	e.l1	u.l1	rw.l1
trend.l1				
prod.l1	1.0000000			
e.l1	5.6015679	-0.73265559		
u.l1	0.7622661	0.04961409		
rw.l1	3.0279450	-0.39917112		
trend.l1	-0.1098165	-0.01391613		
	0.01049672	-0.01101037	-0.02518934	

Weights W:
(This is the loading matrix)

	prod.l1	e.l1	u.l1	rw.l1
trend.l1				
prod.d	0.04945092	-0.8389741		
e.d	-0.04320145	0.4721084	-0.1455891	
u.d	-0.71099542	-11.7211619		
rw.d	-0.02179574	-0.6655943		
	0.6955693	0.14638732	3.625022e-11	

```
> summary(ca.jo(Romania, type =
"trace", ecdet = "trend", K = 2,spec =
"transitory"))
Johansen-Procedure
Test type: trace statistic , with linear trend
in cointegration
```

Eigenvalues (lambda):
[1] 7.329818e-01 6.441662e-01
4.021959e-01 1.670903e-01 -2.577691e-17

Values of test statistic and critical values
of test:

	test 10pct	5pct	1pct
r <= 3	5.30	10.49	12.25 16.26
r <= 2	20.22	22.76	25.32 30.45
r <= 1	50.19	39.06	42.44 48.45
r = 0	88.48	59.14	62.99 70.05

Eigenvectors, normalised to first column:
(These are the cointegration relations)

	prod.l1	e.l1	u.l1	rw.l1
trend.l1				
prod.l1	1.00000000			
e.l1	-0.30056061	1.14171040		
u.l1	0.17068541	0.04906183		
rw.l1	0.17634015	-0.62296427		
trend.l1	-0.03052553	-0.01081926		
	0.008170070	-0.01126230	0.10388828	

Weights W:
(This is the loading matrix)

	prod.l1	e.l1	u.l1	rw.l1
trend.l1				
prod.d	-0.07902084	0.1151782		
e.d	0.04441833	-0.1047434	0.32808259	
u.d	-5.53342117	-9.0910660		
rw.d	-0.27297765	0.7167966		
	0.08963894	-0.051798214	-2.220969e-12	

4. CONCLUSIONS

In this paper we emphasize the utility of the functions and methods contained in the package *vars* within R - software environment for statistical computing. By this package, the researchers have a fairly ease to use environment for conducting VAR, SVAR and SVEC analysis. For empirical experimentation we have a determination of a suitable lag length for the model, stability and causality analysis of Romanian macroeconomic data sets.

5. Acknowledgments

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Biography

Ciprian Alexandru participated in various conferences. His research activity was valued by publishing studies in journals in Romania and Europe, as well as in international databases. He attended a postdoctoral program that implemented the use of R as a tool for analyzing the evolution indices on the stock market.

Nicoleta Caragea participated as a national expert in various projects, workshops and conferences organized by prestigious international institutions with statistics activity. The results of her research activity have been published in many prestigious journals in Romania and abroad and registered in international databases.

Ana Maria Dobre is expert at the National Institute of Statistics within General Department of Social Statistics and Demography. Currently she participates in the development of studies on international migration within which was implemented the use of R. Her research activity consists in participation at conferences and publishing working papers

Annex

	Y	Q	e	prod	rw	u
1	2005	Q1	15.5882	9.3495	6.8189	8.5
2	2005	Q2	15.6141	9.3592	6.8596	7.1
3	2005	Q3	15.5829	9.4338	6.8690	6.1
4	2005	Q4	15.5904	9.4618	6.9441	6.8
5	2006	Q1	15.6167	9.4879	6.9782	7.8
6	2006	Q2	15.6538	9.4910	7.0157	7.0
7	2006	Q3	15.6597	9.5226	7.0309	7.0
8	2006	Q4	15.6078	9.6376	7.1570	7.2
9	2007	Q1	15.6333	9.6524	7.1601	7.0
10	2007	Q2	15.6417	9.6845	7.2262	6.5
11	2007	Q3	15.6437	9.7287	7.2464	6.0
12	2007	Q4	15.6397	9.8032	7.3614	6.1
13	2008	Q1	15.6455	9.8745	7.3784	6.3
14	2008	Q2	15.6546	9.9082	7.4565	5.6
15	2008	Q3	15.6709	9.9352	7.4670	5.4
16	2008	Q4	15.6637	9.9405	7.5429	5.8
17	2009	Q1	15.6492	9.8788	7.5362	6.9
18	2009	Q2	15.6453	9.9006	7.5447	6.3
19	2009	Q3	15.6505	9.8928	7.5330	6.8
20	2009	Q4	15.6234	9.9525	7.5618	7.5
21	2010	Q1	15.6131	9.9434	7.5977	8.1
22	2010	Q2	15.6219	9.9585	7.5817	6.8
23	2010	Q3	15.6128	9.9880	7.5247	6.9
24	2010	Q4	15.6221	9.9947	7.5692	7.3
25	2011	Q1	15.6332	10.0292	7.5947	7.6
26	2011	Q2	15.6380	10.0371	7.6174	7.2
27	2011	Q3	15.6355	10.0640	7.6090	7.2
28	2011	Q4	15.6233	10.0947	7.6451	7.7
29	2012	Q1	15.6378	10.0716	7.6298	7.6
30	2012	Q2	15.6530	10.0823	7.6637	6.9
31	2012	Q3	15.6513	10.1074	7.6633	6.8